NOTE ON REPEATED LETTER-SHUFFLING

J. A. LINDON
Weybridge, Surrey, England

In the February 1970 Word Ways, Walter Penney's cryptographic Puzzle 1 asked the reader to determine the order of a set of cards bearing letters after a single shuffle, if the original order and the new order resulting from two identical shuffles were given. The solution, with its references to trial-and-error, was rather unsatisfactory. In this short note, we suggest a more systematic approach to such problems.

As shuffling is repeated the letters follow one another in cyclic rotation, like fish swimming nose-to-tail in a bowl. Suppose there is just one such group of letters and that our original word (or phrase) has an odd number of letters. Then the matter is quite straightforward. For instance:

Original material	L O S T-E R I C A
After double-shuffle	E A C S I L O R T

Then we may write the double-shuffle operator (DSop) thus: (ELRCSTAOI), which means that in double-shuffle transformations E replaces L, L replaces R, . . . , I (at the end) replaces E (at the beginning).

We now have two ways of finding the required single-shuffle arrangement. The first way is merely to repeat the DS until the missing series returns, as it will when there is a single cycle and an odd number of letters. Here, after five consecutive DS operations, we get the arrangement

T R I E S-A-C O L

which suggests that Erica was lost while mountaineering -- too much snow in that col perhaps.

The second way is to convert our DSop into a SSop, which we can do by spreading the letters and then putting the overlapping ones into the spaces, thus:

DSop	(ELRCSTAOI)
Spreading	(E.L.R.C.S.T.A.O.I)
Interlettering	(ETLAROCIS)

and this is our required SSop. Applying it to the letter series

L O S T-E R I C A

we obtain T R I E S-A-C O L

as we expect.

Here is a similar example for anyone to try.

L O S T-E R I C A

DS O A L S I T C R E

What is the SS form? Or, putting it another way, Erica is lost, poor girl. What does she do?

What happens when we are dealing with an even number of letters? Suppose, for example, that our original material is

N E A T-G I R L

which DS into T A R L E N G I

(Perhaps the exotic name of our neat girl is Tarlengi.) How do we find the SS form here? To put it another way, what ought we to do after performing our exasperated double-shuffle on neat girl Tarlengi's doormat? Here the DSop is (TNIL)(AEGR), which splits as shown into two four-letter cycles. If we treat this as a single series and spread and interletter as before, we get (TANEIGLR) for our SSop. This does indeed give TARLENGI after two applications, but the intermediate form

A N T R I E L G

is nonsense, although a possible intermediate transposition. The point is, a cycle can start anywhere, so, after spreading the letters of the first half, (T.N.I.L.), we can fit in <u>any</u> cyclic transposition of the remaining four (AEGR), and still get a possible intermediate form. And in fact the one we are looking for uses the cyclic transposition (RAEG), giving the SSop (TRNAIELG), which single-shuffles our neat girl into

R I N G-L A T E

obviously the proper thing to do if she is out when you call on her!

Let's return to lost Erica, shall we? Suppose our DS gives us

which Lois (Erica's sister) certainly ought to do if Erica is lost. Well, what is the SS form here? In other words, where is lost Erica? Note that here our original series of letters, the SS form and the DS form all make reasonable sense. Other such cases might be looked for.

The spreading and interlettering of the operator is an idea I owe to Mr J.M. Hendrie, a retired teacher of mathematics living at Streatham Hill, London. Answers to queries can be found in Answers and Solutions at the end of this issue.

QUERY

Can the readers of Word Ways find a locality, village, town or city in a county or parish of the same name in one of the 50 states bearing the same name? Darryl Francis indicates some near-miss solutions. He points out that there is a county of New York in the city of New York in the state of New York. Note that the county is in the city, rather than the other way round. He also points out that Oklahoma City is in the county of Oklahoma in the state of Oklahoma. Further, he indicates that an 1897 map in the Century Atlas of the World lists and shows Oklahoma rather than Oklahoma City. Do any more recent atlases or gazetteers refer to the city in this way? Can Word Ways readers find a more up-to-date reference?

Since currently there are only seven counties in states of the same name, the possibilities are rather limited. If any reader has access to detailed maps of any of the seven counties concerned, perhaps they will be able to find the place that Darryl is looking for. The counties and states are Arkansas, Hawaii, Idaho, Iowa, New York, Oklahoma, and Utah.